Swapskills Pte Ltd

Time Value of Money and Financial Math

TVM 101

"Time value of money" is the value derived from the use of money over time as a result of investment and reinvestment. Time value of money procedures involves computing the present value (PV) or future value (FV) of a set of cash flows. Measuring the PV and FV of an investment's cash flows help to contrast 2 different investment opportunities at a common point in time. While PV is the value at the beginning of an investment horizon, FV is the value at the end of one.

Present Value

Today's value of a cash flow that is to be received at some point in the future. It is computed by the process of "*discounting*" i.e. adjusting future cash flows for interest accrued over the time between the present and future period.

Future Value

The value of a cash flow at a specified in the future which includes interest accrued on the sum invested in a prior period in time. It is computed by the process of "*compounding*" i.e. strapping on interest to today's sum.

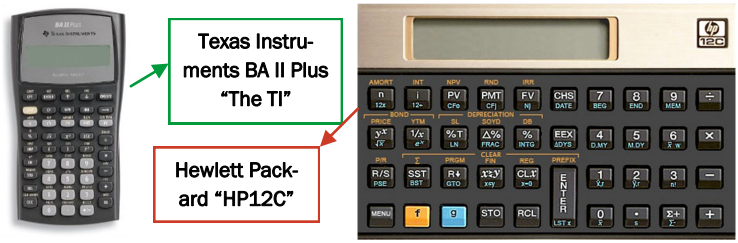
Inside this issue:

Points of interest

- **Pricing any kind of risk involves discounting cash flows**
- **Evaluation of the value of investments / projects at any point in time**
- **Rates may be viewed in several ways**

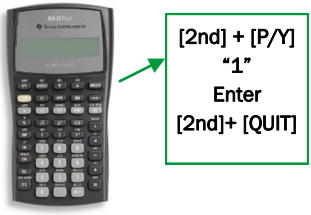
Using a Financial Calculator

The two most popular financial calculators in the industry are the Texas Instruments BAII Plus (and BAII plus professional) and the Hewlett Packard HP12C calculator. We will elucidate the calculations regarding time value of money using both these calculators in this course.



Setting the Periods Per Year

We will perform all computations using an annual compounding period in frame, and suitably vary inputs for different compounding periods. In order to reset the calculator to annual compounding perform the following steps:



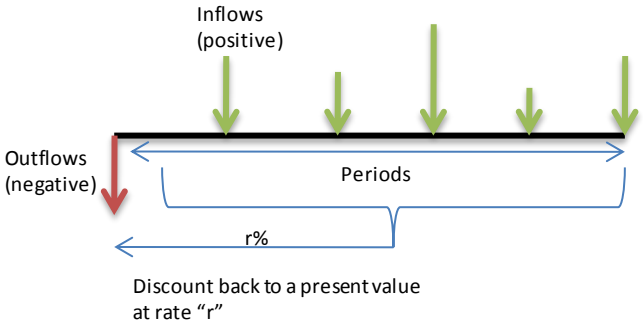
This ensures that we can look at I/Y as interest rate per compounding period and N as the number of compounding periods under analysis.

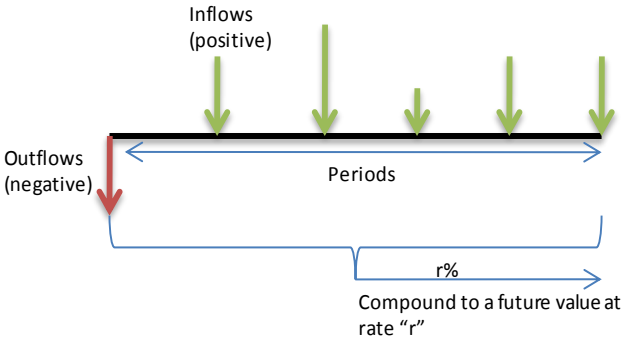
The Important Variables

I/Y	Interest rate per compounding period
PV	Present Value
FV	Future Value
PMT	Periodic constant cash flows
CPT	Compute

Timelines

Timelines are merely diagrams representing the cash flows occurring at different periods in time over the range of the investment's timeframe. The conventions are as below—





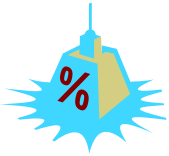
Note The cash flows occur at the end of the period shown on the time-line and that the end of one year is same as the beginning of the next.

An Easy Way to Remember!

The conventions for Time Value of money problems and timeline diagrams vary in every other academic text, but this version is intuitive.

"Arrows going out of the line are outflows and arrows coming into the line are inflows"

Interpreting Interest Rates

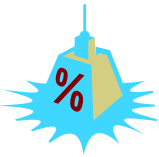


Required Rate of Return
The rate of return the investors in a market expect in compensation to lend out their capital.

Discount Rate
The rate at which cash flows received in the future course of the investment must be *discounted* to derive its value in the present. This is the cost of borrowing.

Opportunity Cost of Capital
It is the amount foregone (expressed in percentage or original investment) by investors when investors choose to prepone their consumption of funds in an investment as opposed to save their capital.

Interest Rates = Risk Free Rate + Risk Premia

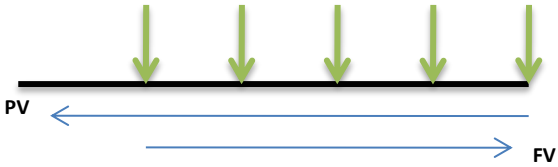


Investors in the capital markets expect to be compensated for every piece of risk they take upon themselves while lending out their capital. Hence it is possible to break up an interest rate (viewed as a rate of return in this context) into the pieces which contribute to investor expectations of return. An interest rate is hence the sum of a theoretical risk free rate (return on a single period loan which has no expectations of inflation) + several risk premia (contribution from taking on additional pieces of risk).

Expected Rate of return =
Real Risk Free Rate +
Inflation premium (risk of falling purchasing power) +
Default Risk (the risk that borrower doesn't honour his commitment) +
Liquidity Risk (The risk of being able to earn less than fair value if an investment must be sold for cash quickly) +
Maturity Risk (The risk of volatility of rates in a long term investment which increases opportunity cost of lending) + etc...

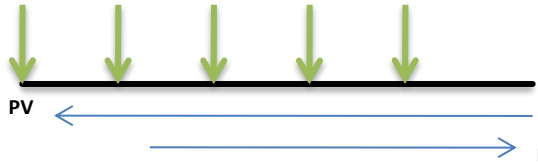
Annuity

An Annuity is a stream of equal cash flows occurring at equal intervals over a given period or simply put a cash flow steam that looks like this!



Ordinary Annuity

In the kind of annuity shown above the cash flows occur at the end of each year / period. Were the cash flows to occur at the beginning, it is called an **Annuity due**!



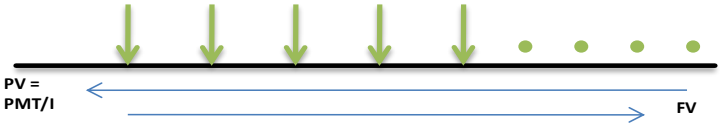
Annuity Due

The formulae for inter-converting between the present and future value of an ordinary annuity (Ao) and an annuity due (Ad) are as below—

PVAd = PVAo (1+I/y)
FVAd = FVAo (1+I/y)

Perpetuity

An infinite term annuity is called a perpetuity. While the future value is infinite (obviously), the present value is obtained as PMT / I.



The concept of Time Value of money is the founding pillars upon which all investment analysis revolves around. Discounting cash flows from a future date to a present value forms the basis of evaluating intrinsic values of stocks, bonds, real-estate, corporate projects, options, futures, private equity investments and merger synergies besides others.

One Stage Examples

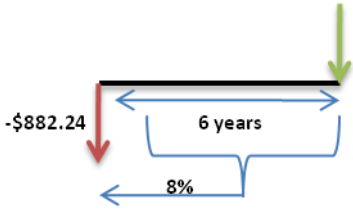
A one stage time value calculation involves crunching only one set of numbers to obtain a result, they may be computing present / future values / yields / periods of compounding / periodic payments given other variables.

Here are some—

Example

Discount a cash flow of $1400 occurring in 6 years time using a discount rate of 8% to find its present value!

Answer:



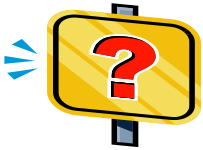
FV=$1400
N=6
I/Y = 8%
PMT=0
CPT PV
PV = -882.24

Present Value and Future value are always of opposing signs. This denotes that an outflow of $882.24 today will compound at 8% in 6 year's time to $1400.

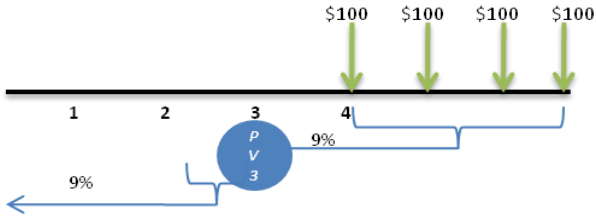
[f][CLEAR FIN][CLX]
6 [n]
1400 [FV]
8 [i]
0 [PMT]
[PV]
PV = -882.2374

And Another One Involving Annuities!

What is the present value of four $100 end of year payments if the first payment is to be received in 4 years time and the interest on the investment is 9%



Answer:



FV=$0
N=4
I/Y = 9%
PMT=100
CPT PV
PV 3= -323.93

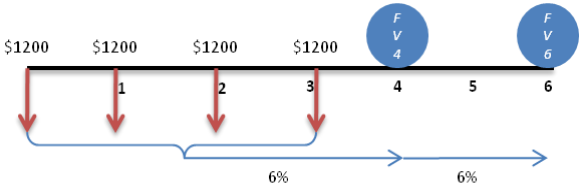
FV = -323.93
N=3
I/Y = 9%
PMT=0
CPT PV
PV 0= 250.13

[f][CLEAR FIN][CLX]
4 [n]
0 [FV]
9 [i]
100 [PMT]
[PV]
PV3 = -323.97

[f][CLEAR FIN][CLX]
3 [n]
323.97 [CHS] [FV]
9 [i]
0 [PMT]
[PV]
PV 0= 250.1643

Here's Another One

If you deposit $1200 in the bank today and at the end of the next 3 years, how much will u have 6 years from today at 6% rate of interest.



For annuity dues, set the calculator to BGN mode—[2nd][BGN] [2nd][SET] [2nd] [QUIT]. PV=$0, N=4, I/Y = 6, PMT=-1200, CPT FV, FV 4= 5564.5116
We'll compound for 2 more periods : FV 6 = 5564.5116(1.06)(1.06) = 6252.29

[f][CLEAR FIN][CLX][g][BEG]; 4[n], 6[i], 0[PV], 1200[CHS][PMT], FV
FV4= 5564.5116 [ENTER] 1.06{x}, 1.06{x} : Hence FV6= 5564.51

Non Annual Compounding Periods

With an increase in the frequency of compounding of the principal amount, the future value is higher and hence increases the effective annual rate of interest. Let's study this using an example—

Rate Adjustments and Annualizing Rates

Compute the present value of a $1000 single sum for an investment horizon of one year using a stated annual rate of 6% with annual, semi-annual, quarterly, monthly and daily compounding, and find the effective annual yield for each compounding frequency.

Try to notice the way we modify the inputs for N and I/Y using multipliers

Compounding Frequency	N	I/Y	FV	PV (Computed)	Effective annual yield
Annual (1)	1	6%	1000	-943.396	(1000/943.396) - 1 = 6%
Semi Annual (2)	1 x 2	(6/2)%	1000	-942.596	(1000/942.596) - 1 = 6.09%
Quaterly (4)	1 x 4	(6/4)%	1000	-942.184	(1000/942.184) - 1 = 6.136%
Monthly (12)	1 x 12	(6/12)%	1000	-941.905	(1000/941.905) - 1 = 6.168%
Daily (365)	1 x 365	(6/365)%	1000	-941.769	(1000/941.769) - 1 = 6.183%

We can hence derive the following relation:

Effective annual yield = $(1+i/m)^m - 1$, where i = stated rate.

Rule of Thumb on Compounding Frequency

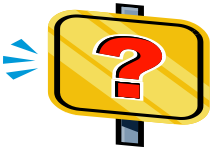
Greater compounding periods means **Greater** interest upon interest earned and hence a **greater** effective annual yield.

Example

A bond pays $1000 at maturity in 5 years and pays a 7% semi-annual coupon. Calculate the present value of the cash flows if the yield to maturity is 8%

FV=$1000; N=5*2 ;I/Y = 8/2%; PMT=35 ;CPT PV ;PV = -959.445

And Another One



Sarah puts in $100 twice each year into a savings account which pays 8% compounded semi-annually for a period of 4 years. Joanne wishes to purchase the same item that Sarah's saving for but would however like to make payments each quarter into her savings account which pays her 7% compounded quarterly. How much must Joanne deposit each quarter?

PV=0
N=4*2
I/Y = 8/2%
PMT=-100
CPT FV
FV = 921.4226

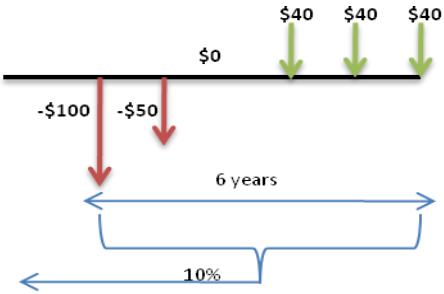
The first step would be to compute how much Sarah had in 4 years time!

FV=921.4226
PV = 0
N=4*4
I/Y = 7/4%
CPT PMT
PMT = -50.4014

Joanne has the same amount in 4 years time, but invests quarterly. Hence she'd have to deposit $50.40 each quarter.

Uneven Cash Flow Streams

So how to you evaluate the present value of this cash flow stream? You could either evaluate the present value of each individual cash flow of course, but is there an easier way?



Such cash flows occur frequently in project finance. Some projects require multiple rounds of investment before gains / profits are realized.

Using a Cash Flow Worksheet

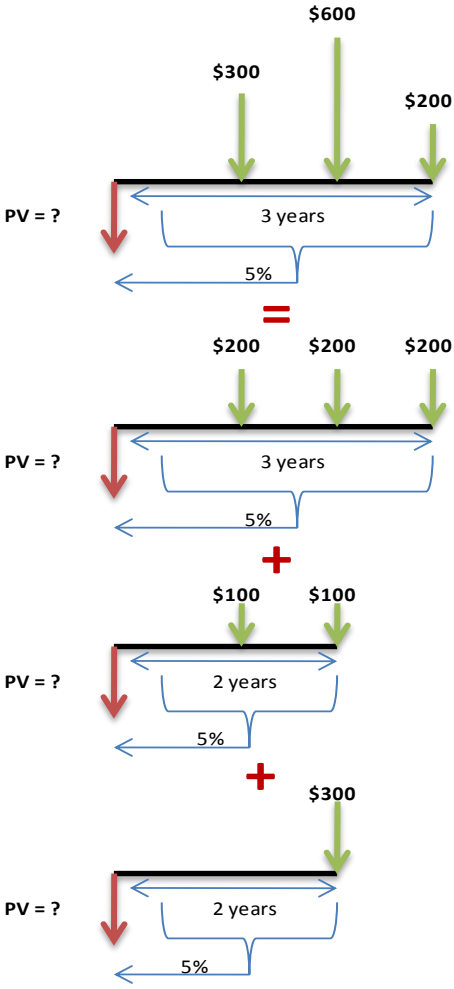
It is also possible to compute the present value (or net present value) of an uneven cash flow stream using the cash flow worksheet and the net present value function of the calculator. Here's how it goes—

Keystrokes	What it means?	Inputs	Frequency of cash flows
[CF] [2nd] [Clr work]	Clear CF worksheet memory	CF0 = 0	
0 [ENTER]	Initial Cash outlay	CF0 = 0	
[↓] 100 [+/-] [ENTER]	1st period cash	C01 = -100	F01= 1
[↓][↓] 50 [+/-] [ENTER]	2nd period cash	C02 = -50	F02 = 1
[↓][↓] 0 [ENTER]	3rd period cash	C03 = 0	F03 = 1
[↓][↓] 40 [ENTER]	Subsequent 3 cash flows	C04=40	F04 = 3
[NPV] 10 [ENTER]	Discount @ 10%	I = 10	
[↓] [CPT]	Calculate NPV	NPV = -57.4951	

Keystrokes	What it means?	Display
[f] [FIN] [f][REG]	Clears memory registers	0.0000
0 [g][CF0]	Initial Cash outlay	0.0000
100[CHS][g][CFj]	1st period cash	-100.0000
50[CHS][g][CFj]	2nd period cash	-50.0000
0[g][CFj]	3rd period cash	0.0000
40[g][CFj]	4th period cash	40.0000
40[g][CFj]	5th period cash	40.0000
40[g][CFj]	6th period cash	40.0000
10[i]	Discount @ 10%	10.0000
[f][NPV]	Calculate NPV	-57.4951

Additivity Principle of Cash flows

Two sums of cash can be added up together (at the same point in time), and hence cash flows are additive in nature. Using this to our advantage we can break up uneven cash flow streams into simple combinations of single sums (or) annuities. Take a look—



We want to evaluate the present value of this cash flow stream!

Evaluate this part as a 3 year ordinary annuity!

Evaluate this part as a 2 year ordinary annuity!

Evaluate this part as a single sum of cash receivable in 2 years!

Test your understanding!

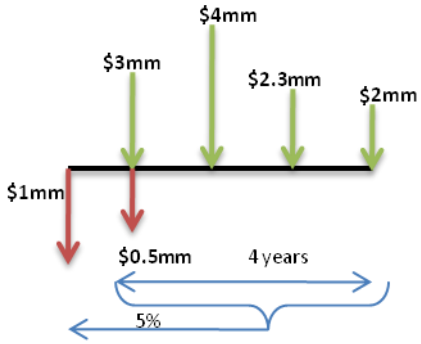
1. Petro-Jupiter (PJ) is a oil company undertaking an exploration project. They have managed to secure a ring-fence type financing from a premier bank. In this type of financing structure, the bank will extend $1mm and $0.5mm in financing for the first couple of years. PJ expects to discover oil at the end of the first year which they expect to translate to revenues of $3m. They also expect to earn revenues of $4m, $2.3m and $2m in the next three years. Compute the value added to the firm by undertaking this project. The annual cost of financing is 5%.

2. Consider a stock with dividends that are expected to grow at 20% for the next 4 years after which they will resume normal growth of 5% per year indefinitely. The last dividend paid was $1.00 and the cost of common equity to the firm is 10%. Calculate the value of this stock.
(**Hint**: the value of the stock is the PV of all future dividends paid and value of a constant growth perpetuity is given by D1/k-g where D is the first dividend paid under sustainable long-term constant growth, k the cost of equity to the firm and g the constant rate of growth of dividends)

3. The Osbournes plan to take 3 holidays, one each year. The first holiday they will take 6 years from today, the second 8 years from today and a third 10 years from today. They currently spend $10,000 on any holiday but they expect inflation to increase this cost at 4% each year on an average. They will contribute to an account to save for these holidays that will earn 8% each year. What equal contribution must they make today and every year until their first holiday (6 contributions) in order to have saved enough for all three holidays at that time? They pay for the holidays when taken.

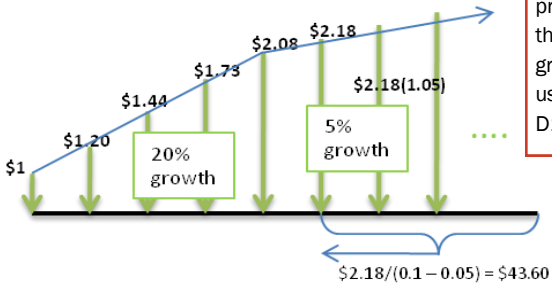
Answers

1. This is simply a question on finding the net present value of the uneven cash flow stream shown below—

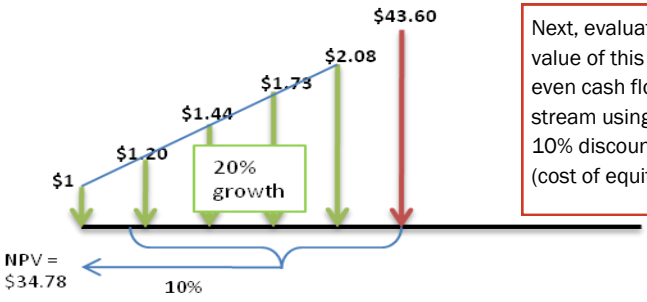


CF0 = -1
CF1 = 2.5
CF2 = 4
CF3 = 2.3
CF4 = 2
I = 5%
CPT NPV
NPV = $8.6413m

2. The cash flows are as show below—

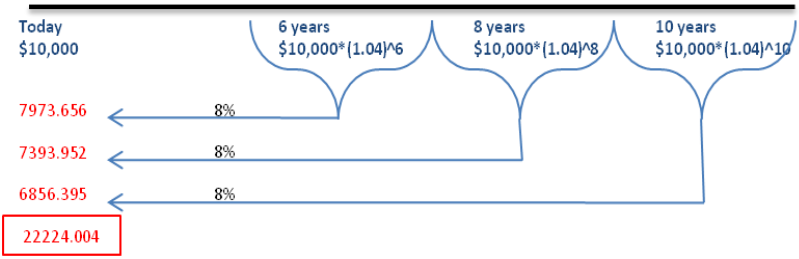


First evaluate the present value of the constant growth perpetuity using the formula D1/k-g



Next, evaluate the value of this un-even cash flow stream using a 10% discount rate (cost of equity)

3. The cash flows are as below—



The PV of all 3 holidays is $22224.004 = PV of the 6 period annuity due. [2nd][BGN][2nd][SET]

PV = 22224.004 , i/y = 8, N = 6, CPT PMT = -4806.9614

Notes

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